Heim’s Mass Formula (1982)

Original Text by Burkhard Heim
for the Programming of his Mass Formula

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On the Description of Elementary Particles

(Selected Results)

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A) Invariants of Possible Basic Patterns (Multiplets)

Symbols:

k	Configuration number, $k = 0$: no ponderable particle (no rest mass). For ponderable particles only $k = 1$ and $k = 2$ possible, <u>not</u> $k > 2$. k is a metrical index number.
ε	so-called "time-helicity". Refering to the R_4 $\varepsilon = +1$ or $\varepsilon = -1$ decides whether it concerns an R_4-structure or the mirror-symmetrical anti-structure ($\varepsilon = -1$).
G	the number of quasi-corpuscular internal sub-constituents of structural kind.
b_i	Symbol for these $1 \le i \le G$ internal sub-constituents of an elementary particle.
B	Baryon number
P	double isospin $P = 2s$.
$\underline{P}_{1,2}$	locations in P-interval, where multiplets appear multiplied (doubled).
I	number of components y of an isospin-multiplet, i.e $1 \le y \le I$.(Ele2, S.279 u 284)
Q	double space-spin $Q = 2J$.
$\underline{Q}$	value of Q at $\underline{P}_{1,2}$.
$\kappa(\lambda)$	"Doublet-number", which distinguishes between several doublets by $\kappa(\lambda) = 0$ or $\kappa(\lambda) = 1$.
Λ	upper limit of κ-interval $1 \le \lambda \le \Lambda$.
C	Structure-distributor, identical with sign of charge of the strangeness quantum number.
q_x	electrical charge quantum number with sign of the component x of the isospinmultiplet.
q	amount of charge quantum number $q = \lvert q_x \rvert$.

<u>Uniforme Description of Quantum Numbers by k and ε</u>

$$\left.\begin{aligned}
G &= k + 1 \\
B &= k - 1 \\
\underline{P}_1 &= 2 - k \\
\underline{P}_2 &= 2k - 1 \\
I &= P + 1 \,, \qquad 0 \le P \le G \\
Q(P) &= k - 1 \\
\underline{Q}(\underline{P}) &= 2k - 1 \\
\kappa(\lambda) &= (1 - \delta_{1\lambda})\, \delta_{1P} \,, \qquad 1 \le \lambda \le \Lambda = 4 - k \\
C &= 2(P\varepsilon_P + Q\varepsilon_Q)(k - 1 + \kappa)/[k\,(1 + \kappa)] \\
\varepsilon_{P,Q} &= \varepsilon \cos \alpha_{P,Q}
\end{aligned}\right\} \text{(I)}$$

$$\left.\begin{aligned}\alpha_P &= \pi Q(\kappa + \tbinom{P}{2})\\ \alpha_Q &= \pi Q[Q(k-1) + \tbinom{P}{2}]\\ 2q_x &= (P-2x)[1-\kappa Q(2-k)] + \varepsilon[k-1-(1+\kappa)Q(2-k)] + C\,,\ x = y-1\,,\ 0 \le x \le P\\ q &= |q_x|\end{aligned}\right\} \text{(II)}$$

Possible configurations $k = 1, k = 2$ with $\varepsilon = \pm 1$

Possible Multiplets of Basic States

Multiplet x_ν of serial number ν for $\varepsilon = +1$ and anti,multiplet $\bar{x}_\nu$ with $\varepsilon = -1$.

General Representation: $\bar{x}_\nu(\varepsilon B,\varepsilon P,\varepsilon Q,\varepsilon\kappa)_\varepsilon C(q_0,\ldots,q_P)$

Mesons: $k = 1$, $G = 2$ (quark?), $B = 0$, $0 \le P \le 2$, i.e from singulet $I = 1$ to triplet $I = 3$.
$\underline{Q} = 0$, $\underline{Q} = 1$, $\Lambda(k = 1) = 3$, $\kappa(1) = 0$, $\kappa(2) = \kappa(3) = 1$

Baryons: $k = 2$, $G = 3$ (quark?), $B = 1$, $0 \le P \le 3$ from singulet $I = 1$ to quartet $I = 4$,
$\underline{Q} = 1$, $\underline{P}_1 = 0$, $\underline{P}_2 = 3$, $\underline{Q} = 3$, $\Lambda(k = 2) = 2$, $\kappa(1) = 0$, $\kappa(2) = 1$

Possible multipletts for $\varepsilon = +1$:

$k = 1$:

$x_1\,(0000)0(0) \equiv (\eta)$
$x_2\,(0110)0(0,-1) \equiv (e_0, e^-)$, (is the existence of e_0 possible ?)
$x_3\,(0111)0(-1,-1) \equiv x_3\,(0111)0(-1) \equiv (\mu^-)$ Pseudo-singulet }(III)
$x_4\,(0101)+1(+1,0) \equiv (K^+, K^0)$
$x_5\,(0200)0(+1,0,-1) \equiv x_5\,(0200)0(\pm1,0) \equiv (\pi^\pm, \pi^0)$ Anti-triplet to itself?

$k = 2$:

$x_6\,(1010)\text{-}1(0) \equiv (\Lambda)$
$x_7\,(1030)\text{-}3(-1) \equiv (\Omega^-)$
$x_8\,(1110)0(+1,0) \equiv (p, n)$
$x_9\,(1111)\text{-}2(0,-1) \equiv (\Xi^0, \Xi^-)$ }(IV)
$x_{10}\,(1210)\text{-}1(+1,0,-1) \equiv (\Sigma^+, \Sigma^0, \Sigma^-)$
$x_{11}\,(1310)\text{-}2(+1,0,-1,-2) \equiv (o^+, o^0, o^-, o^{--})$, (existence possible ?)
$x_{12}\,(1330)0(+2,+1,0,-1) \equiv (\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$, (thinkable as a basic state ?)

Abbreviations:

$$\left.\begin{aligned}\eta &= \pi/(\pi^4+4)^{1/4}\\ \eta_{qk} &= \pi/[\pi^4 + (4+k)q^4]^{1/4}\\ \vartheta &= 5\,\eta + 2\sqrt{\eta} + 1\\ A_1 &= \sqrt{\eta_{11}}\,(1-\sqrt{\eta_{11}})/(1+\sqrt{\eta_{11}})\\ A_2 &= \sqrt{\eta_{12}}\,(1-\sqrt{\eta_{12}})/(1+\sqrt{\eta_{12}})\end{aligned}\right. \qquad \text{(V)}$$

Planck's constant: $\hbar = h/2\pi$, Speed of light: $c = (\varepsilon_0\mu_0)^{-1/2}$,
Wave-resistance of empty space R_3 (electro-magnetic): $R_- = c\mu_0$, with ε_0 and μ_0 constants of influence and induction.

Electrical elementary charge: $e_\pm = 3C_\pm$ with

$$C_\pm = \pm\sqrt{2\vartheta\hbar / R_-}\ /(4\pi^2)$$ (possibly electr. quark-charge ?)

Finestructure-constant: $\alpha\sqrt{(1-\alpha^2)} = 9\vartheta(1 - A_1A_2)/(2\pi)^5$, $\alpha > 0$.

Solution: $\alpha_{(+)}$ (positive branch) and $\alpha_{(-)}$ (negative branch).

Numerical: $\alpha_{(+)}{}^{-1} = 137{,}03596147$
$\alpha_{(-)}{}^{-1} = 1{,}00001363$

What is the meaning of that strong coupling $\alpha_{(-)}$?
Abbreviation: $\alpha_{(+)} = \alpha$, $\alpha_{(-)} = ß \approx 137\,\alpha$.

B) Mass-Spectrum of Basic Patterns and its Resonances

Used constants of nature and pure numbers:

Planck's constant: $\hbar = h/2\pi = 1{,}0545887 \times 10^{-34}$ J s,
Speed of light: $c = 2{,}99792458 \times 10^{8}$ m s^{-1},
Newton's constant of gravitation: $\gamma = 6{,}6732 \times 10^{-11}$ N m^2 kg^{-2},
Constant of influence: $\varepsilon_0 = 8{,}8542 \times 10^{-12}$ A sV^{-1} m^{-1},
Constant of induction: $\mu_0 = 1{,}2566 \times 10^{-6}$ A^{-1} s V m^{-1},
Vacuum wave-resistance: $R_- = (\mu_0/\varepsilon_0)^{1/2} = 376{,}73037659$ V A^{-1};

Derived constant of nature (mass-element):

$$\mu = \sqrt[4]{\pi}\sqrt[3]{3\pi\gamma\hbar s_0}\sqrt{c\hbar/3\gamma}\,(cs_0)^{-1}\ ,\quad s_0 = 1\ [\mathrm{m}]\ \text{(gauge factor)} \tag{VI}$$

Basis of natural logarithms: $e = 2{,}71828183$,
Number $\pi = 3{,}1415926535$,
Geometrical constant: $\xi = 1{,}61803399$,
[Limes of the "creation-selector"] $\lim_{n\to\infty} a_n : a_{n-1} = \xi$ by the series $a_n = a_{n-1} + a_{n-2}$
(represented by $\xi = (1 + \sqrt{5})/2$).

Auxiliary functions:

$$\eta = \pi/(\pi^4 + 4)^{1/4} \tag{VII}$$

$$\left.\begin{aligned} t &= 1 - 2/3\,\xi\,\eta^2(1 - \sqrt{\eta}) \\ \alpha_+ &= t\,(\eta^2\,\eta^{1/3})^{-1} - 1 \\ \alpha_- &= t\,(\eta\eta^{1/3})^{-1} - 1 \end{aligned}\right\}\text{(VIII)}$$

Quantum numbers by (A):

$$\eta_{qk} = \pi/[\pi^4 + (4+k)q^4]^{1/4}$$

$$\left.\begin{aligned} N_1 &= \alpha_1\,, \\ N_2 &= (2/3)\,\alpha_2\,, \\ N_3 &= 2\,\alpha_3\,, \\ &\text{with} \\ \alpha_1 &= \tfrac{1}{2}(1+\sqrt{\eta_{qk}})\,, \\ \alpha_2 &= 1/\,\eta_{qk}\,, \\ \alpha_3 &= e^{(k-1)}/k - q\,\{\alpha/3\,(1+\sqrt{\eta_{qk}})\,(\xi/\eta_{qk}^{\,2})^{(2k+1)}\,\eta_{qk}^{\,3} + \\ &\quad + [\eta(1,1)/(e\,\eta_{qk})]\,(2\xi\eta_{qk})^k\,[(1-\sqrt{\eta_{qk}})/(1+\sqrt{\eta_{qk}})]^2\,\} \end{aligned}\right\} \quad \text{(IX)}$$

Invariants of metrical steps-structure (abbreviation $s = k^2 + 1$):

$$\begin{aligned} Q_1 &= 3\cdot 2^{\,s-2}\,, \\ Q_2 &= 2^s - 1\,, \\ Q_3 &= 2^s + 2(-1)^k\,, \\ Q_4 &= 2^{s-1} - 1\,. \end{aligned} \quad \text{(X)}$$

Fourfold R_3-construct $1 \le j \le 4$. Q_j = const with respect to time t.
Parameter of occupation $n_j = n_j(t)$ caused radioactive decay.
Mass elements of occupations of the configurations zones j are $\mu\alpha_+$.

Further auxiliary functions of zones occupations:

$$\begin{aligned} K &= n_1^{\,2}\,(1+n_1)^2N_1 + n_2\,(2n_2^2 + 3n_2 + 1)N_2 + n_3\,(1+n_3)N_3 + 4n_4\,, \\ \underline{G} &= Q_1^2(1+Q_1)^2N_1 + Q_2(2Q_2^2 + 3Q_2 + 1)N_2 + Q_3(1+Q_3)N_3 + 4Q_4\,, \\ H &= 2n_1Q_1[1 + 3\,(n_1 + Q_1 + n_1Q_1) + 2(n_1^2 + Q_1^2)]N_1 + 6n_2Q_2(1 + n_2 + Q_2)N_2 + \\ &\quad + 2n_3Q_3N_3 \end{aligned} \quad \text{(XI)}$$

$$\begin{aligned} \Phi &= 3\,P/(\pi\sqrt{\eta_{qk}})\,(1 - \alpha_-/\alpha_+)\,(P+Q)(-1)^{P+Q}\,[1 - \alpha/3 + \pi/2\,(k-1)\,3^{1-q/2}\,] \\ &\quad \cdot \{1 + 2k\kappa/(3\,\eta^2)\,\xi[1 + \xi^2(P-Q)(\pi^2 - q)]\}\,/\,[1 + (\,4\,\xi\tbinom{P}{2}/k)(\xi/6)^q] \\ &\quad \cdot [\,2\sqrt{(\eta_{11}\eta_{qk})} + q\eta^2\,(k-1)]\,[1 + (4\pi\alpha)/(\eta\sqrt{\eta})]\,[1 + Q(1-\kappa)(2-k)n_1/Q_1] \\ &\quad + 4\,(1 - \alpha_-/\alpha_+)\alpha(P+Q)/\xi^2\ + 4\,q\alpha_-/\alpha_+ \end{aligned}$$

<u>Uniform Mass spectrum:</u>

$$M = \mu\alpha_+\,(K + \underline{G} + H + \Phi) \qquad \text{[kg]} \qquad \text{(XII)}$$

Not each quadruple n_j yields a real mass! To the selection rule: in the fourfold R_3-construct $1 \le j \le 4$ configurations zones $n(j=1)$, $m(j=2)$, $p(j=3)$, $\sigma(j=4)$.
Increase of occupation with metrical structure elements:
<u>Central zone</u> n cubic,
<u>Internal zone</u> m quadratic
<u>Meso-zone</u> p linear (continuation to the empty space R_3)
<u>External zone</u> σ selective.

Principle of increase of the configurations zones:

$$n_4 + Q_4 \le (n_3 + Q_3)\alpha_3 \le (n_2 + Q_2)^2\,\alpha_2 \le (n_1 + Q_1)^3\alpha_1 \qquad \text{(XIII)}$$

<u>Selection rule for the Occupation of Configuration Zones</u>

$$(n_1 + Q_1)^3\alpha_1 + (n_2 + Q_2)^2\,\alpha_2 + (n_3 + Q_3)\alpha_3 + \exp[(1-2k)(n_4 + Q_4)/3Q_4] + \mathrm{i}F(\Gamma) = \text{(XIV)}$$
$$= W_{\nu x}\{1 + [1 - Q(2-k)(1-\kappa)][a_{\nu x}N/(N+2) + b_{\nu x}\sqrt{N(N-2)}\,]\}$$

$$W_{\nu x} = \mathrm{g}(qk)\, w_{\nu x}\,,$$

Basis rise: $\quad \mathrm{g}(qk) = Q_1^{\,3}\alpha_1 + Q_2^{\,2}\,\alpha_2 + Q_3\alpha_3 + \exp[(1-2k)/3] \quad$ for $n_j = 0$. (XV)

Structure power of the discussed state $w_{\nu x} = (kPQ\kappa)_\varepsilon C(q_x)$ as component x of multiplets ν is:

$$\begin{aligned} w_{\nu x} = {} & \{(1-Q)[A_{11} - P(A_{12} + A_{13}q\kappa/\eta_{qk}) - \tbinom{P}{2}(A_{14} - A_{15}q/\eta_{qk})] + \kappa Q\eta_{qk}A_{16}\}^{2-k} + \\ & + \{(q-1)A_{21} + (1-P)A_{22} + \tbinom{P}{2}[A_{23} - q_x\eta_{qk}(1 + A_{24}(1 + q_x))^{-1}A_{25}] + \\ & + \kappa(A_{26} + q\,\eta_{qk}^{\,2}A_{31}) + \tbinom{Q}{3}\eta_{qk}A_{32} + \tbinom{P}{3}[A_{33}\,q^3(q_x - (-1)^q)/(3-q) + \\ & + \frac{\varepsilon(P-Q)\eta^{(q-1)q/4}}{8 - A_{66}\,q(q-1)}\,(1 - q(2-q)\;A_{34}^{\,(1-q_x)}\;A_{35}/\eta_{qk})\;\eta_{qk}/\eta^2 - A_{36}]\}^{k-1}\,. \end{aligned} \qquad \text{(XVI)}$$

$$w(1) = (1-Q)[A_{11} - P(A_{12} + A_{13}q\kappa/\eta_{qk}) - \tbinom{P}{2}(A_{14} - A_{15}q/\eta_{qk})] + \kappa Q\,\eta_{qk}A_{16} \qquad \text{(XVII)}$$

and

$$\begin{aligned} w(2) = {} & (q-1)A_{21} + (1-P)A_{22} + \tbinom{P}{2}[A_{23} - A_{25}q_x\eta_{qk}(1 + A_{24}(1+q_x))^{-1}] + \\ & + \kappa(A_{26} + q\,\eta_{qk}^{\,2}A_{31}) + \tbinom{Q}{3}\eta_{qk}A_{32} + \tbinom{P}{3}\{A_{33}\,q^3(q_x - (-1)^q)/(3-q) + \\ & + \frac{\varepsilon(P-Q)\eta^{(q-1)q/4}}{8 - A_{66}\,q(q-1)}\,[1 - q(2-q)\,A_{34}^{\,(1-q_x)}\,A_{35}/\eta_{qk}]\;\eta_{qk}/\eta^2 - A_{36}\} \end{aligned} \qquad \text{(XVIII)}$$

In $\quad w_{\nu x} = [w(1)]^{2-k} + [w(2)]^{k-1} \qquad$ (XIX)

can become $w(2) = 0$ for single sets of quantum numbers at $k = 1$ or $w(1) = 0$ at $k = 2$, which leads to terms 0^0, which but must have always have the value 1 as parts of structure power. Therefore it is recommended for programming to complete $w(1)$ and $w(2)$ by the numerical non-relevant summands $k-1$ and $2-k$. Since always $w(1) \ne -1$ and $w(2) \ne -1$ remain, but only $k = 1$ or $k = 2$ is possible, the actually terms in the expression

$$w_{\nu x}(k) = [k - 1 + w(1)]^{2-k} + [2 - k + w(2)]^{k-1}$$

do no more appear. By this correction it is evident that for mesonical structures $w_{\nu x}(k=1) = 1 + w(1)$ and for barionical structures $w_{\nu x}(k=2) = 1 + w(2)$ holds.

As a basis of resonance holds: $a_{vx} = A_{41}(1 + a_n a_q)/k$ (XX)

with: $a_n = P A_{42}[1 - \kappa A_{43}(1 + A_{44}(-\alpha)^{2-k} A_{45}{}^{k-1})*$
$*(1 - \kappa Q A_{46}(2 - k)) - A_{51}(k-1)(1-\kappa)]$ (XXI)

and $a_q = 1 - q A_{52}(1 - 2\kappa A_{53}{}^{k})[1 + q_x(3 - q_x)(k-1)(1-\kappa)/6]$. (XXII)

Resonance grid is:

$$b_{vx} = \{A_{54} A_{55}{}^{k-1}[1 + P A_{56}(1 - \kappa A_{61} A_{62}{}^{1-k})(1 + q A_{63}(1 + \kappa A_{64}))] *$$
$$*(1 - k^{-1}(A_{65}(q + k - 1))^{2-k}\binom{P}{2}(1 - \binom{P}{3}))\}/[k^P(1 + P + Q + \kappa\eta^{2-q})] \quad \text{(XXIII).}$$

The coefficients A_{rs} can be seen as elements of the quadratic coefficient matrix $\hat{A} = (A_{rs})_6$ with Im $A_{rs} \neq A_{sr}$ and Im$A_{rs} = 0$.

Proposal for the determination of matrix elements (reduction to π, e and ξ):

$$A_{11} = (\xi^2 \pi e)^2 (1 - 4\pi\alpha^2)/2\eta^2,$$
$$A_{12} = 2\pi e \xi^2 (\vartheta/24 - e\pi\eta\alpha^2/9)$$
$$A_{13} = 3(4 + \eta\alpha)[(1 - (\eta^2/5)(1 - \sqrt{\eta})^2/(1 + \sqrt{\eta})^2]$$
$$A_{14} = [1 + 3\eta(2\eta\alpha - e^2\xi)(1 - \sqrt{\eta})^2/\{(1 + \sqrt{\eta})^2 4\xi\}]/\alpha$$

$$A_{15} = e^2(1 - 2e\alpha^2/\eta)/3$$
$$A_{16} = (\pi e)^2[1 + \alpha(1 + 6\alpha/\pi)/5\eta]$$
$$A_{21} = 2(e\alpha/2\eta)^2(1 - \alpha/2\xi^2)$$
$$A_{22} = \xi[1 - \xi(\alpha\xi/\eta^2)^2]/12$$

$$A_{23} = (\eta^2 + 6\xi\alpha^2)/e$$
$$A_{24} = 2\xi^2/3\eta$$
$$A_{25} = \xi(\pi e)^2(1 - \beta^2)$$
$$A_{26} = 2\{1 - [\pi(e\xi\alpha)^2\sqrt{\eta}]/2\}/e\xi^2$$

$$A_{31} = (\pi e\alpha)^2[1 - (\pi e)^2(1 - \beta^2)]$$
$$A_{32} = \xi^2[1 + (2e\alpha/\eta)^2]/6$$
$$A_{33} = (\pi e\xi\alpha)^2[1 - 2\pi(e\xi)^2(1 - \beta^2)]$$
$$A_{34} = \eta\sqrt{2\pi\eta}$$

(XXIV)

$$A_{35} = 3\alpha/e\xi^2$$
$$A_{36} = [1 - \pi e(\xi e)^2(1 - \beta^2)]^{-1}$$
$$A_{41} = \{\xi[2 + (\xi\alpha)^2] - 2\beta\}/(2\beta - \alpha)$$
$$A_{42} = [\pi\xi^2\eta(\beta - 3\alpha)]/2$$

$$A_{43} = \xi/2$$
$$A_{44} = 2(\eta/\xi)^2$$
$$A_{45} = (3\beta - \alpha)/6\xi$$
$$A_{46} = \pi e/\xi\eta - e\eta^2\alpha/2$$

$A_{51} = (2\alpha + 1)^2$
$A_{52} = 6\alpha/\eta^2$
$A_{53} = (\xi/\eta)^3$
$A_{54} = \alpha(\beta - \alpha)\sqrt{(3/2)}$

$A_{55} = \xi^3$
$A_{56} = (\xi/\eta)^4$
$A_{61} = \pi\xi(2\beta - \alpha)/12\beta$
$A_{62} = \pi^2(\beta - 2\alpha)/12$

$A_{63} = (\sqrt{\eta})/9$
$A_{64} = \pi/3\eta$
$A_{65} = \pi/3\xi$
$A_{66} = \xi\eta$

The order of resonance (resonance allocation) $N \geq 0$ (positive int) selects the admitted quadruple n_j with $1 \leq j \leq 4$ aus. With the abbreviation

$$f(N) = [1 - Q(2 - k)(1 - \kappa)][a_{\nu x} N/(N + 2) + b_{\nu x} \sqrt{N(N-2)}\,] \qquad \text{(XXV)}$$

follows that the unknown function $F(\Gamma) = 0$ for all $N \neq 1$ bleibt (right side is real).
In the case of $N = 0$ is $f = 0$, so that

$$(n_1 + Q_1)^3\alpha_1 + (n_2 + Q_2)^2\,\alpha_2 + (n_3 + Q_3)\,\alpha_3 + \exp[(1 - 2k)(n_4 + Q_4)/3Q_4] = W_{\nu x} \qquad \text{(XXVI)}$$

describes the n_j of the state $x_{\nu x}$ and hence the mass $M_0(\nu x)$ of the component x of the multiplet x_ν. The $N \geq 2$ assign $x_{\nu x}$ to a spectrum of occupation-parameter quadruples and with that, according to the mass-formula, resonance-masses $M_N(\nu x)$ (for each component $x_{\nu x}$ a spectrum of masses). In the case of $N = 1$ no spectral term. Here is not $f(N) \geq 0$, $f(1)$ is complex.

Real part: $(\underline{n}_1 + Q_1)^3\alpha_1 + (\underline{n}_2 + Q_2)^2\,\alpha_2 + (\underline{n}_3 + Q_3)\,\alpha_3 + \exp[(1 - 2k)(\underline{n}_4 + Q_4)/3Q_4]$
$= W_{\nu x}\{1 + [1 - Q(2 - k)(1 - \kappa)]a_{\nu x}/3\}$ (XVII)

Imaginary part: $F(\Gamma) = W_{\nu x}[1 - Q(2 - k)(1 - \kappa)]b_{\nu x}$. (XXVIII)

The $\underline{n}_j$ and $F(\Gamma)$ are somehow related with N to the complete bandwidths Γ. Also there must be a connection $Q_N = Q(N)$ between doubled spin quantum-number Q and N.
How could this connection be like?

If $N = 1$ is excluded, then $F = 0$, and the real relationship:

$$(n_1 + Q_1)^3\alpha_1 + (n_2 + Q_2)^2\,\alpha_2 + (n_3 + Q_3)\,\alpha_3 + \exp[(1 - 2k)(n_4 + Q_4)/3Q_4]$$
$$= W_{\nu x}\,(1 + f) \qquad \text{(XXIX)}$$

has to be discussed. Generally $f > 0$ for $N \geq 2$ and $f = 0$ for $N = 0$.
But in the case of the multiplets x_2 is $f = 0$ for all $N \geq 0$, since only here is $Q(2 - k)(1 - \kappa) = 1$. Electrons according to this image can not be stimulated !

For a numerical evaluation of $W_{\nu x}$, $a_{\nu x}$, $b_{\nu x}$ and $\Phi_{\nu x}$ (quantum number function in mass spectrum M) not $Q_N = Q(N)$, but use $Q = Q(0)$ of x_ν . For the evaluation of n_j the principle of increase of the occupations of configuration zones is considered. First determine the right side $W_{\nu x}(1 + \mathrm{f}(N)) = W_1$ numerically for an order of resonance $N = 0$ or $N \geq 2$. Determine according to the selection rule the maximal cubic number K_1^3 whose product with α_1 is contained in W_1 .
Then insert $W_1 - \alpha_1 K_1^3 = W_2 \geq 0$ into:

$$(n_2 + Q_2)^2\, \alpha_2 + (n_3 + Q_3)\, \alpha_3 + \exp[(1 - 2k)(n_4 + Q_4)/3Q_4] = W_2 \qquad \text{(XXX).}$$

Now maximal quadratic number K_2^2 such, that $\alpha_2 K_2^2$ is still a factor of W_2 , i.e. $W_2 - \alpha_2 K_2^2 = W_3 \geq 0$. Accordingly in

$$(n_3 + Q_3)\, \alpha_3 + \exp[(1 - 2k)(n_4 + Q_4)/3Q_4] = W_3 \qquad \text{(XXXI)}$$

determine maximal number K_3 in the way $W_3 - \alpha_3 K_3 = W_4 \geq 0$.

Three possibilities for W_4 : (a): $W_4 = 0$,
(b): $0 < W_4 \leq 1$,
(c): $W_4 > 1$.

General case (b): $\ln W_4 \leq 0$ and $K_4(2k - 1) = -3Q_4 \ln W_4$.
In case of (c) it is $\ln W_4 > 0$ and $K < 0$.
This is impossible, since always $n_j + Q_j \geq 0$ has to be.
According to $n_4 + Q_4 \leq (n_3 + Q_3)\alpha_3$ of the principle of rise K_3 will be lowered by 1 and $\alpha_3 K_3$ is added to $K_4 < 0$, so that a new value $K_4 \geq 0$ will be generated, which requires $K_3 > 0$, since in that case $K_3 = 0$. This dilatation can not happen because of the quadratic rise of $j = 2$, so that this order of resonance N does not exist for $x_{\nu x}$ (forbidden term).

In the case (a) $W_4 \to 0$ would have as a consequence the divergence $K_4 \to \infty$, but this is impossible according to $K_4 \leq \alpha_3 K_3$ (particularly there are no diverging self-potentials).
For that reason will be calculated in case of (a) the maximal value $K_4 = \alpha_3 K_3$.
From the computed K_j it follows $n_j = K_j - Q_j$. Beside $n_j \geq 0$ also $n_j < 0$ is possible, but it holds always $K_j \geq 0$, i.e. $n_j \geq -Q_j$. The quadruple n_j determined in that way will be inserted with $\Phi_{\nu x}$ in the spectrum of masses, which numerically yields $M_N(\nu x)$ as a spectral-term of mass-spectrum at $x_{\nu x}$.

Note: The K_j are always integers. But in the case of the evaluation of K_4 generally decimal figures will occur. In case of the decimal places $.99 \ldots \overline{99}$ one has to use the identity $.99 \ldots \overline{99} = 1$. But if the series of decimal places is different from this value, then one has not to round up. The decimal places are to cut off , since the K_j are the numbers of structure entities.

(Remark OP 2006: Those wrong decimal places are a result of using programming languages with insufficient numerical accuracy). Numerical accuracy has to be greater than the number of digits used.

Limits of Resonance Spectra

General construction-principle of Configuration zones:

$$n_4 + Q_4 \leq (n_3 + Q_3)\alpha_3 \,,$$
$$\alpha_3\,(n_3 + Q_3)(1 + n_3 + Q_3) \leq 2\alpha_2(n_2 + Q_2)^2 \,, \qquad \text{(XXXII)}$$
$$\alpha_2\,(n_2 + Q_2)[2(n_2 + Q_2)^2 + 3(n_2 + Q_2) + 1] \leq 6\alpha_1\,(n_1 + Q_1)^3 \,.$$

If by the increase of N between two zones equality is reached, then $n_j + Q_j \rightarrow 0$ in j, while $j - 1$ will be raised by 1 to $n_{j-1} + Q_{j-1} + 1$. The stimulation takes place "from outside to the interior". Always $n_j + Q_j \geq 0$ is an integer, since they are the numbers of structure entities.
Empty-space-condition: $n_j = -Q_{j\ j}$, but $(n_j)_{\max} = L_j < \infty$ (no diverging self-energy potentials).
Intervals $-Q_j \leq n_j \leq L_j < \infty$ cause $0 \leq N \leq L < \infty$ of resonance-order.
With $M_0(\nu x) = M_0$ holds

$$4\mu\alpha_+\alpha_1\,(L_1 + Q_1)^3 = [2(P + 1)]^{2-k} M_0\, G \qquad \text{(XXXIII)}$$

with $G = k + 1$ and from that by the construction-principle

$$\alpha_2(L_2 + Q_2)[2(L_2 + Q_2)^2 + 3(L_2 + Q_2) + 1] \leq 6\alpha_1\,(L_1 + Q_1)^3 \,,$$
$$\alpha_3(L_3 + Q_3)(1 + L_3 + Q_3) \leq 2\alpha_2(L_2 + Q_2)^2 \,, \qquad \text{(XXXIV)}$$
$$L_4 + Q_4 \leq (L_3 + Q_3)\alpha_3 \,.$$

For L implicitly the resonance-order is:

$$(L_1 + Q_1)^3\alpha_1 + (L_2 + Q_2)^2\,\alpha_2 + (L_3 + Q_3)\,\alpha_3 + \exp[(1 - 2k)(L_4 + Q_4)/(3Q_4)] =$$
$$= W_{\nu x}\,[1 + f(L)] \qquad \text{(XXXV)}$$

Also in the evaluation of L_j and L do not round up, but cut off decimal digits! The L_j which are obtained by the construction-principle, yield the absolute maximal masses $M_{\max}$, and the quadruples, which are obtained from the L, yield the real limit-terms $M_L < M_{\max}$, which are to stimulate secondaryly with $(M_{\max} - M_L)c^2$ and then reach $M_{\max}$.

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Changelog

Version	Date	Changes
0.23	2009-10-12	Added missing brackets in Fi (XI), line 2 and 3