

Dear Sirs:

Using the equation (IX) for α_3 of script of 1982 indicated down:

$$\alpha_3(q, k) := \frac{e^{k-1}}{k} - q \cdot \left[\frac{\alpha}{3} \cdot (1 + \sqrt{\eta_-(q, k)}) \cdot \left(\frac{\xi}{\eta_-(q, k)^2} \right)^{(2 \cdot k + 1)} \cdot \eta_-(q, k)^3 + \left(\frac{\eta_-(1, 1)}{e \cdot \eta_-(q, k)} \right) \cdot (2 \cdot \xi \cdot \eta_-(q, k))^k \cdot \left[\frac{(1 - \sqrt{\eta_-(q, k)})}{(1 + \sqrt{\eta_-(q, k)})} \right]^2 \right]$$

With α given by:

$$A1 := \sqrt{\eta_-(1, 1)} \cdot \frac{(1 - \sqrt{\eta_-(1, 1)})}{(1 + \sqrt{\eta_-(1, 1)})}$$

$$A2 := \sqrt{\eta_-(1, 2)} \cdot \frac{(1 - \sqrt{\eta_-(1, 2)})}{(1 + \sqrt{\eta_-(1, 2)})}$$

$$D := 9 \cdot \theta \cdot \frac{(1 - A1 \cdot A2)}{(2 \cdot \pi)^5}$$

$$\alpha := \sqrt{\frac{-1 + \sqrt{1 - 4 \cdot D^2}}{-2}}$$

$$\alpha = 7.297354597564723 \times 10^{-3}$$

$$\frac{1}{\alpha} = 1.3703596099519688 \times 10^2$$

The following values are obtained:

$$k := 1$$

$$q := 0..k$$

$$N1(q, k) =$$

$1 \cdot 10^0$
$9.968812705531683 \cdot 10^{-1}$

$$N2(q, k) =$$

$6.666666666666666 \cdot 10^{-1}$
$6.750617425258186 \cdot 10^{-1}$

$$N3(q, k) =$$

$2 \cdot 10^0$
$1.9573175917804535 \cdot 10^0$

$$k := 2$$

$$q := 0..k$$

$$N1(q, k) =$$

$1 \cdot 10^0$
$9.962780882675572 \cdot 10^{-1}$
$9.589182514626359 \cdot 10^{-1}$

$$N2(q, k) =$$

$6.666666666666666 \cdot 10^{-1}$
$6.767036958714698 \cdot 10^{-1}$
$7.913672787769991 \cdot 10^{-1}$

$$N3(q, k) =$$

$2.718281828459045 \cdot 10^0$
$2.5988192179329896 \cdot 10^0$
$2.007462491238667 \cdot 10^0$

On the other hand using the equations of script of 1989? (4a)# and (8c)# we obtain:

$$\alpha_p := \sqrt[6]{\eta} \cdot \frac{\left[1 - \theta \cdot \left[\frac{2}{\eta} \cdot \frac{(1 - \sqrt{\eta})}{(1 + \sqrt{\eta})} \right]^2 \cdot \sqrt{2 \cdot \eta} \right]}{\eta^2} - 1 \quad \alpha_p = 1.832212396977306 \times 10^{-2}$$

$$\alpha_n := -1 + \eta \cdot (1 + \alpha_p) \quad \alpha_n = 8.12835374722542 \times 10^{-3}$$

$$N3(k, q) := \frac{2}{k} \cdot e^{(k-1) \cdot \left[1 - \pi \cdot \frac{1 - \eta_-(q, k)}{1 + \sqrt{\eta_-(q, 1)}} \cdot \left[1 - u \cdot \frac{\eta_-(q, 1)}{\theta_-(q, 1)} \cdot \left(1 - \frac{\alpha_n}{\alpha_p} \right) \cdot (1 - \sqrt{\eta})^2 \right] \right]} \cdot \frac{\omega}{c \cdot u} \cdot (1 - \sqrt{\eta})^2 \cdot \left[2 \cdot \frac{c \cdot u^2}{\omega \cdot \theta} \cdot \frac{1 + \sqrt{\eta_-(q, 1)}}{(1 - \eta)} - 1 \right]$$

k := 1

q := 0..k

N1(k, q) =

1 · 10 ⁰
9.968812705531683 · 10 ⁻¹

N2(k, q) =

6.666666666666666 · 10 ⁻¹
6.750617425258187 · 10 ⁻¹

N3(k, q) =

1.9571855627151193 · 10 ⁰
1.9573176662909733 · 10 ⁰

N4(k, q) =

4 · 10 ⁰
4 · 10 ⁰

N5(k, q) =

1.157733756335647 · 10 ⁰
1.157733756335647 · 10 ⁰

N6(k, q) =

1.6444579190054325 · 10 ⁻⁶
1.6444579190054325 · 10 ⁻⁶

k := 2

q := 0..k

N1(k, q) =

1 · 10 ⁰
9.962780882675572 · 10 ⁻¹
9.589182514626359 · 10 ⁻¹

N2(k, q) =

6.666666666666666 · 10 ⁻¹
6.767036958714699 · 10 ⁻¹
7.913672787769991 · 10 ⁻¹

N3(k, q) =

2.66009097502545 · 10 ⁰
2.5988192395716236 · 10 ⁰
2.0592841745803985 · 10 ⁰

N4(k, q) =

2 · 10 ⁰
4 · 10 ⁰
6 · 10 ⁰

N5(k, q) =

1.157733756335647 · 10 ⁰
1.73239192592923 · 10 ⁰
7.673188427276104 · 10 ¹

N6(k, q) =

-1.0521531495463794 · 10 ⁻¹
2.525567092143019 · 10 ⁻²
1.557266567974983 · 10 ⁻¹

As i mentioned before in another email, observing the second term of the equation (IX), we noticed that it appears multiplied by q . Now if we propose that there is a lost factor q in the second term within the exponential function of the equation (8c)# and correcting the same we obtain:

$$N3(k, q) := \frac{2}{k} \cdot e^{(k-1) \cdot \left[1 - \pi \cdot \frac{1 - \eta_-(q, k)}{1 + \sqrt{\eta_-(q, 1)}} \cdot \left[1 - u \cdot \frac{\eta_-(q, 1)}{\theta_-(q, 1)} \cdot \left(1 - \frac{\alpha n}{\alpha p} \right) \cdot (1 - \sqrt{\eta})^2 \right] \right]} \cdot \frac{\omega}{c \cdot u} \cdot (1 - \sqrt{\eta})^2 \cdot \left[2 \cdot \frac{c \cdot u^2}{\omega \cdot \theta} \cdot \frac{1 + \sqrt{\eta_-(q, 1)}}{(1 - \eta)} - 1 \right] \cdot q$$

$k := 1$

$q := 0..k$

$N1(k, q) =$

$1 \cdot 10^0$
$9.968812705531683 \cdot 10^{-1}$

$N2(k, q) =$

$6.666666666666666 \cdot 10^{-1}$
$6.750617425258187 \cdot 10^{-1}$

$N3(k, q) =$

$2 \cdot 10^0$
$1.9573176662909733 \cdot 10^0$

$N4(k, q) =$

$4 \cdot 10^0$
$4 \cdot 10^0$

$N5(k, q) =$

$1.157733756335647 \cdot 10^0$
$1.157733756335647 \cdot 10^0$

$N6(k, q) =$

$1.6444579190054325 \cdot 10^{-6}$
$1.6444579190054325 \cdot 10^{-6}$

$k := 2$

$q := 0..k$

$N1(k, q) =$

$1 \cdot 10^0$
$9.962780882675572 \cdot 10^{-1}$
$9.589182514626359 \cdot 10^{-1}$

$N2(k, q) =$

$6.666666666666666 \cdot 10^{-1}$
$6.767036958714699 \cdot 10^{-1}$
$7.913672787769991 \cdot 10^{-1}$

$N3(k, q) =$

$2.718281828459045 \cdot 10^0$
$2.5988192395716236 \cdot 10^0$
$2.0167757114412734 \cdot 10^0$

$N4(k, q) =$

$2 \cdot 10^0$
$4 \cdot 10^0$
$6 \cdot 10^0$

$N5(k, q) =$

$1.157733756335647 \cdot 10^0$
$1.73239192592923 \cdot 10^0$
$7.673188427276104 \cdot 10^1$

$N6(k, q) =$

$-1.0521531495463794 \cdot 10^{-1}$
$2.525567092143019 \cdot 10^{-2}$
$1.557266567974983 \cdot 10^{-1}$

Then $N3(1,0)$, $N3(2,0)$ and $N3(2,1)$ are corrected. Now appear the following questions:

It is possible that really that factor q is lost?

Must $N3(2,2) = 2.007...$ of 1982 be equal to $N3(2,2) = 2.016...$ of 1989?

It is possible that this difference is of some form included in the function $\phi(p, \sigma)$?

The discrepancies in $N5$ and $N6(2, q)$ must to other values of αp and αn ?

These discrepancies, I do not believe, to be due to a lack of precision in the calculation program.

Best regards

Javier Mazzone