B. Heim : Different versions of the constants
Mathcad, versión 14.0.0.163 [revisión 702091438]
2009-11-18

$$\eta := \frac{\pi}{\sqrt[4]{\pi^4 + 4}}$$

$$\xi := \frac{1 + \sqrt{5}}{2}$$

$$\theta := 5\cdot\eta + 2\cdot\sqrt{\eta} + 1$$

$$s := \frac{-\eta}{4\cdot\xi}$$

$$\eta\_(q,k) := \frac{\pi}{\sqrt[4]{\pi^4 + (4 + k)\cdot q^4}}$$

$$\theta\_(q,k) := 1 + 5\cdot\eta\_(q,k) + 2\cdot\sqrt{\eta\_(q,k)}$$

$$A1 := \sqrt{\eta\_(1,1)}\cdot\frac{\left(1 - \sqrt{\eta\_(1,1)}\right)}{\left(1 + \sqrt{\eta\_(1,1)}\right)}$$

$$A1b := \eta\_(1,1)^s\cdot\frac{\left(1 - \sqrt{\eta\_(1,1)}\right)}{\left(1 + \sqrt{\eta\_(1,1)}\right)}$$

$$A2 := \sqrt{\eta\_(1,2)}\cdot\frac{\left(1 - \sqrt{\eta\_(1,2)}\right)}{\left(1 + \sqrt{\eta\_(1,2)}\right)}$$

$$K\alpha 1989a := 1 - (1 + \eta\_(2,2))\cdot\left[\frac{1 - \sqrt{\eta}}{\eta\_(1,1)\cdot\left(1 + \sqrt{\eta}\right)}\right]^2$$

$$K\alpha 1989b := 1 - \frac{(1 + \eta\_(2,2))}{\eta\cdot\eta\_(1,1)\cdot\eta\_(1,2)}\cdot\left(\frac{1 - \sqrt{\eta}}{1 + \sqrt{\eta}}\right)^2$$

$$A2b := \eta\_(1,2)^s\cdot\frac{\left(1 - \sqrt{\eta\_(1,2)}\right)}{\left(1 + \sqrt{\eta\_(1,2)}\right)}$$

$$D := 9\cdot\theta\cdot\frac{(1 - A1\cdot A2)}{(2\cdot\pi)^5}$$

$$dd1989a := \frac{(2\cdot\pi)^5}{9\cdot\theta\cdot K\alpha 1989a}$$

$$dd1989b := \frac{(2\cdot\pi)^5}{9\cdot\theta\cdot K\alpha 1989b}$$

$$Db := 9\cdot\theta\cdot\frac{(1 - A1b\cdot A2b)}{(2\cdot\pi)^5}$$

$$\alpha 1982 := \sqrt{\frac{-1 + \sqrt{1 - 4\cdot D^2}}{-2}}$$

$$\alpha 1989a := \sqrt{\frac{2}{dd1989a^2\cdot\left(1 + \sqrt{1 - \frac{4}{dd1989a^2}}\right)}}$$

$$\alpha 1989b := \sqrt{\frac{2}{dd1989b^2\cdot\left(1 + \sqrt{1 - \frac{4}{dd1989b^2}}\right)}}$$

$$\alpha 1992 := \sqrt{\frac{-1 + \sqrt{1 - 4\cdot Db^2}}{-2}}$$

$$\beta 1982 := \sqrt{\frac{-1 - \sqrt{1 - 4\cdot D^2}}{-2}}$$

$$\beta 1989a := \sqrt{\frac{2}{dd1989a^2\cdot\left(1 - \sqrt{1 - \frac{4}{dd1989a^2}}\right)}}$$

$$\beta 1989b := \sqrt{\frac{2}{dd1989b^2\cdot\left(1 - \sqrt{1 - \frac{4}{dd1989b^2}}\right)}}$$

$$\beta 1992 := \sqrt{\frac{-1 - \sqrt{1 - 4\cdot Db^2}}{-2}}$$

$$t := 1 - \frac{2}{3}\cdot\xi\cdot\eta^2\cdot\left(1 - \sqrt{\eta}\right)$$

$$\alpha p1982 := t\cdot\left(\eta^{2}\cdot\eta^{\frac{1}{3}}\right)^{-1} - 1$$

$$\alpha n1982 := t\cdot\left(\eta\cdot\eta^{\frac{1}{3}}\right)^{-1} - 1$$

$$\alpha p1989a := \sqrt[6]{\eta}\cdot\frac{\left[1 - \theta\cdot\left[\frac{2}{\eta}\cdot\frac{\left(1-\sqrt{\eta}\right)}{\left(1+\sqrt{\eta}\right)}\right]^{2}\cdot\sqrt{2\cdot\eta}\right]}{\eta^{2}} - 1$$

$$\alpha n1989a := -1 + \eta\cdot(1 + \alpha p1989a)$$

$$\alpha p1989b := \sqrt[6]{\eta}\cdot\frac{\left[1 - \theta\cdot\left[\frac{2}{\eta}\cdot\frac{\left(1-\sqrt{\eta}\right)}{\left(1+\sqrt{\eta}\right)}\right]^{2}\cdot\sqrt{2\cdot\eta}\right]}{\eta^{2}} - 1$$

$$\alpha n1989b := -1 + \eta\cdot(1 + \alpha p1989b)$$

Version 1982

$$\alpha p1982 = 1.8322115078164725 \times 10^{-2}$$

$$\alpha n1982 = 8.128344944625443 \times 10^{-3}$$

$$\alpha 1982 = 7.297354597564723 \times 10^{-3}$$

$$\frac{1}{\alpha 1982} = 1.3703596099519688 \times 10^{2}$$

$$\beta 1982 = 9.999733739534655 \times 10^{-1}$$

$$\frac{1}{\beta 1982} = 1.0000266267554998 \times 10^{0}$$

Version 1989a (Script of 43 pages)

$$\alpha p1989a = 1.8322123969773063 \times 10^{-2}$$

$$\alpha n1989a = 8.128353747225425 \times 10^{-3}$$

$$\alpha 1989a = 7.297351511666251 \times 10^{-3}$$

$$\frac{1}{\alpha 1989a} = 1.3703601894486013 \times 10^{2}$$

$$\beta 1989a = 9.999733739762247 \times 10^{-1}$$

$$\frac{1}{\beta 1989a} = 1.0000266267327393 \times 10^{0}$$

Version 1989b (Script of 57 pages)

$$\alpha p1989b = 1.8322123969773063 \times 10^{-2}$$

$$\alpha n1989b = 8.128353747225425 \times 10^{-3}$$

$$\alpha 1989b = 7.297350415494955 \times 10^{-3}$$

$$\frac{1}{\alpha 1989b} = 1.37036039529722 \times 10^{2}$$

$$\beta 1989b = 9.999733739838948 \times 10^{-1}$$

$$\frac{1}{\beta 1989b} = 1.0000266267250686 \times 10^{0}$$

Auerbach-Ludwiger JSE 1992 HT document "D" eq. (8.14)

$$\alpha 1992 = 7.297353075631085 \times 10^{-3}$$

$$\frac{1}{\alpha 1992} = 1.3703598957537335 \times 10^{2}$$

$$\beta 1992 = 9.999733739645718 \times 10^{-1}$$

$$\frac{1}{\beta 1992} = 1.0000266267443927 \times 10^{0}$$